Averages - Key Notes

**Averages, Mixtures and Alligation Aptitude basics, practice questions, answers and explanations**

- Average= $(x_1+x_2+x_3+......x_n)/n$
  where $x_1, x_2, x_3,$ are quantities.

- Weighted Average= $(n_1x_1+n_2x_2+n_3x_3+.....n_kx_k) / n_1+n_2+n_3+......n_k$
  where $x_1, x_2, x_3.... x_k$ are the quality factors
  $n_1, n_2, n_3,........n_k$ are the quantity factors

  Eg: If the average height of boys= 172cms and that of girls=154 cms, then find average height of the class with 18 boys and 12 girls?

  Here n_1 and n_2 are no. of boys and girls (Quantity factor)
  x_1 and x_2 are average heights (Quality factor)

- **Mixtures**
  For mixtures, Average, $x'= (n_1x_1+n_2x_{2)/}(n_1+n_2)$

- **Alligation:**

n1   :   n2

x1   x2

x'
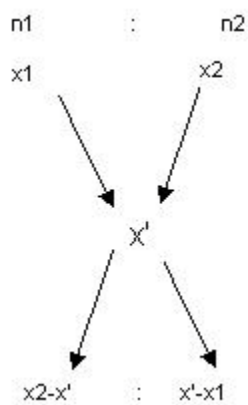
x2-x'   :   x'-x1

=> $n_1/n_2 = (x_2-x')/x'-x_1$

Exercise questions

1. The average wages of a worker during a fortnight comprising 15 consecutive working days was Rs.90 per day. During the first 7 days, his average wages was Rs.87/day and the average wages during the last 7 days was Rs.92 /day. What was his wage on the 8th day?

a)83

b) 92

c) 90

d)97

2. The average of 5 quantities is 6. The average of 3 of them is 8. What is the average of the remaining two numbers?

a) 6.5   b) 4   c) 3   d) 3.5

3. The average temperature on Wednesday, Thursday and Friday was 250. The average temperature on Thursday, Friday and Saturday was 240. If the temperature on Saturday was 270, what was the temperature on Wednesday?

a) 240

b) 210

c) 270

d) 300

4. The average age of a group of 12 students is 20years. If 4 more students join the group, the average age increases by 1 year. The average age of the new students is

a) 24

b) 26

c) 23

d) 22

5. When a student weighing 45 kgs left a class, the average weight of the remaining 59 students increased by 200g. What is the average weight of the remaining 59 students?

a) 57 kgs

b) 56.8 kgs

c) 58.2 kgs

d)52.2 kgs

6. The average of 5 quantities is 10 and the average of 3 of them is 9. What is the average of the remaining 2?

a) 11

b) 12

c) 11.5

d) 12.5

7. The average age of a family of 5 members is 20 years. If the age of the youngest member be 10 years then what was the average age of the family at the time of the birth of the youngest member?

a) 13.5

b) 14

c) 15

d) 12.5

8. Average cost of 5 apples and 4 mangoes is Rs.36. The average cost of 7 apples and 8 mangoes is Rs.48. Find the total cost of 24 apples and 24 mangoes.

a) Rs.1044

b) RS.2088

c) Rs.720

d) Rs.324

9. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be:

a) Rs. 169.50
b) Rs. 170
c) Rs. 175.50
d) Rs. 18

10. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

a) 1/3

b) 1/4
c) 1/5
d) 1/7

Answer Key

1.d; 2.c; 3.d; 4.a; 5.a; 6.c; 7.d; 8.b; 9.c; 10.c

Concepts and Theory

Important Formula and Equations

- Average= Sum of observations/ Number of observations
- The average of odd numbers from **1 to n is   = [Last odd no. + 1]/ 2.**

- The average of even numbers from **1 to n is = [Last even no. + 2]/2.**

Name	Equation or description
Arithmetic mean	$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + \cdots + x_n)$$
Median	The middle value that separates the higher half from the lower half of the data set
Geometric median	A rotation invariant extension of the median for points in Rn
Mode	The most frequent value in the data set
Geometric mean	$$\left(\prod_{i=1}^{n} x_i \right)^{\frac{1}{n}} = \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$
Harmonic mean	$$\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$
Quadratic mean (or RMS)	$$\sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2} = \sqrt{\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}}$$
Generalized mean	$$\sqrt[p]{\frac{1}{n} \cdot \sum_{i=1}^{n} x_i^p}$$

Weighted mean	$$\frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w_1 + w_2 + \cdots + w_n}$$
Truncated mean	The arithmetic mean of data values after a certain number or proportion of the highest and lowest data values have been discarded
Interquartile mean	A special case of the truncated mean, using the interquartile range
Midrange	$$\frac{\max x + \min x}{2}$$
Winsorized mean	Similar to the truncated mean, but, rather than deleting the extreme values, they are set equal to the largest and smallest values that remain
Annualization	$$\left[\prod (1 + R_i)^{t_i} \right]^{1/\sum t_i} - 1$$

o The Average of any number of quantities is sum of their quantities by the number of quantities (n).
   Average=Sum of quantities/n

o If there are two types of items say A and B , A has m number of sub items and B has n number of sum items then the average of A and B is (Am+Bn)/(m+n)

o If a vehicle travels from one place to another at a speed of a kmph but returns at the speed of b kmph then its average speed during the whole journey is 2ab/(a+b) kmph.

o Out of three numbers, first number is x times of the second number and y times of the third number. If the average of all the three numbers is z then the first number is   3xyz /(xy+x+y)

o Let the average age of men and women in a town be x years and the average age of women be y years and the average age of men be z years. Then the number of men in that town is _N(x-y)/(z-y)
   if N indicates the total number of men and women of the town.

o The average age of N persons is x years. If one new person joins them. Then the average age is increased by y years. Then the age of new comer is x + (1 + N) y years.

o The average age of N persons is x years. If M persons joins them, the average age is increased by y years then the average age of newcomers is x+(1+(N/M)) y years

o The average age of N persons is x years. If M persons joins them, the average age is decreased by y years then the average age of new comers is x-(1+(N/M)) y years

o The average age of N persons is x years. If M persons left, then the average age is increased by y years, then the average age ofoutgoing persons is x+(1-(N/M)) y years.

o The average age of N persons is x years. If M persons left, then the average age is decreased by y years. Then the average age of outgoing persons is x-(1-(N/M)) y years

o In a group of N persons whose average age is increased by y years when a person of x years is replaced by a new man. Then the age of new comer is x + Ny years.

o The average temperature of Sunday, Monday, Tuesday and Wednesday was X^o C. The average temperature for Monday, Tuesday, Wednesday and Thursday was Y^o C. If the temperature on Thursday is a^o C then the temperature on Sunday (b^o C) can be given as b^o C = No of days (X -Y) + a
Here No. of days = 4.

Exercise Questions

1) Find the average of first 40 natural numbers.

a. 20.5

b. 18

c. 19.5

d. 19

2) Find the average of all the numbers between 6 and 34 which are divisible by 5

    a. 18

    b. 20

    c. 24

    d. 30

3) The average of 2,7,6 and x is 5 and the average of 18,1,6,x and y is 10. What is the value of y?

    a. 5

    b. 10

    c. 20

    d. 30

4) The average of 7 consecutive numbers is 20. The largest of these numbers is

    a. 20

    b. 22

    c. 23

    d. 24

5) Nine persons went to a hotel for taking their meals. Eight of them spent Rs 12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine. What was the total money spent by them?

    a. 117

    b. 180

c. 150

d. 200

6) In seven given numbers, the average of first four numbers is 4 and that of the last four numbers is also 4. If the average of these seven numbers is 3, the fourth number is

a. 3

b. 4

c. 7

d. 11

7) The average weight of 29 students is 28 kg. By the admission of a new student, the average weight is reduced to 27.8 kg. The weight of the new student is

a. 22 kg

b. 21.6 kg

c. 22.4 kg

d. 21 kg

8) The average age of a committee of 8 members is 40 years. A member aged 55 years retired and his place was taken by another member aged 39 years . The average age of present committee is;

a. 39 years

b. 38 years

c. 36 years

d. 35 years

9) Eight persons participated in a shooting competition. The top score in the competition is 85 points. Had the top score been 92 points instead of 85 points, the average score would have been 84. Find the number of points actually scored in the competition.

   a. 645

   b. 655

   c. 665

   d. 636

10) Find the average of all even numbers upto 75.

   a. 35

   b. 36

   c. 38

   d. 34

11) The average mark of a class of twenty students is 64. If three students whose marks are 32,28 and 34 are removed , then find the approximate average mark of the remaining students of the class.

   a. 71

   b. 74

   c. 57

   d. 70

12) The number of students in the three sections of a class are in the ratio 2:3:4. The average marks scored in each of these sections is in the ratio 4:3:1. By what percent is the average mark of the second section more than the class average?

   a. 23.27%

   b. 28.57%

c. 32.38%

d. 36.74%

13) The average age of 40 students is 8 years. If the age of teacher is also included , then their average age increases by half a year. What is the age of the teacher?

a. 45 years

b. 48.5 years

c. 28.5 years

d. 26.5 years

14) Eight kilograms of rice costing Rs. 16 per kg is mixed with four kilograms of rice costing Rs. 22 per kg. What is the average price of the mixture?

a. 20

b. 18

c. 16

d. 19

15) How many kilograms of tea powder costing Rs. 31 per kg be mixed with 36 kilograms of tea powder costing Rs. 43 per kg, such that the mixture when sold at Rs. 44 per kg gives profit of 10%?

a. 12

b. 15

c. 20

d. 10

16) A solution of 66 litres contains milk and water in the ratio 7:x. If four litres of water is added to the solution, the ratio becomes 3:2, find the value of x?

   a. 8

   b. 5

   c. 3

   d. 4

17) A single refined oil can contains 20% impurities. After double – refining it contains 4% impurities. How much of double- refined oil can be obtained from 30 litres of single refined oil?

   a. 24.0 litres

   b. 24.8 litres

   c. 25.0 litres

   d. 25.5 litres

18) A mixture of 20 kg of spirit and water contains 10% water. How much water must be added to this mixture to raise the percentage of water to 25%

   a. 4 kg

   b. 5 kg

   c. 8 kg

   d. 30 k

19) A can contains a mixture of two liquids A and B in the ratio 7:5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7:9. How many litres of liquid A was contained by the can initially?

   a. 25

   b. 21

c. 20

d. 10

20) Equal weights of two alloys containing tin, copper and lead in the ratio 3:2:7 and 4:11:3 are melted and mixed together. What is the ratio of tin, copper and lead in the resultant alloy?

a. 41:81:37

b. 33:91:81

c. 17:28:27

d. 51:86:89

Answer & Explanations

1. Exp. Sum of first n natural numbers = $\frac{n(n+1)}{2}$

   So, sum of first 40 natural numbers = 40*41/2 = 820

   Required average =820/40 =20.5

2. Exp. Multiples of 5 between 6 and 34 are 10,15,20,25,30

   Average = (10+15+20+25+30)/5 =5(10+30)/2*5 = 40/2 =20

3. Exp. We have : (2+7+6+x)/4 = 5 or 15+x =20 or x = 5

   Also (18+1+6+x+y)/5 = 10, 25+5+y = 50, y =20

4. Exp. Let the first number be x, Then the last number is (x+6)

   Average = $\frac{x+(x+1)+(x+2)+(x+3)+(x+3)+(x+4)+(x+5)+(x+6)}{7}$ = 20

   7x + 21 = 20*7 = 140, 7x =119, x = 17

The largest number   =x+6 =17+6 = 23

5. Exp. Let the total expenditure be x, Then the average = x/9,

   8*12+ [x/9+8] =x or [x-x/9] =104.

   8x/9 = 104, x =104*9/8 =117.

6. Exp. Let the fourth number be x, Then , $\underline{\text{First three} + x}$ = 4

   $\qquad\qquad\qquad\qquad\qquad\qquad 4$

   First three +x = 16, $\underline{x + \text{last three}}$ = 4, x + last three = 16,

   $\qquad\qquad\qquad\qquad 4$

   $\underline{(\text{First three} + x) + \text{last three}}$ = 3, First three +x + last three = 7*3 = 21

   $\qquad\qquad\qquad 7$

   16 + (16 -x) = 21, x = 32 – 21 = 11

7. Exp . The total weight of 29 students = 29* 28

   The total weight of 30 students = 30* 27.8

   Weight of the new student = (30*27.8 – 29*28)

   = 834 - 812 = 22

8. Exp. Total age of the committee = 40*8 = 320,

   Total age when a member is retired

   and a new one was joined = 320-55 +39, =304

   Average age of present committee = 304/8 = 38.

9. Exp. Let the actual number of points scored be x,

   Then [x + (92 –85)] /8 = 84, (x + 7) /8 = 84 , x = (84*8)- 7,

   = 672 - 7 = 665

10. Exp. Average of all even numbers upto 75 = [35/2* (first even number + greatest even number before 75)]/35 = ½ *(2 +74) = 76/2 = 38.

11. Exp. Total mark of 20 students = 64*20 = 1280,

Total mark after the removal of 3 students = 1280 –(32 +28+34)

= 1280 – 94 = 1186

Approximate average mark = 1186/(20-3) = 1186/ 17 = 70

12. Exp. Let the number of students be 2x, 3x, 4x.

Let the average marks be 4y,3y,y.

Average mark of class = (8xy+9xy+4xy)/(2x+3x+4x) =21xy/9x=7y/3

Percentage difference = (3y - 7y/3)/7y/3 *100 = 28.57%

13. Exp. Total age of 40 students = 40*8 = 320

Let the age of the the teacher be x, Then (320+x)/41 = 8+1/2 =8 ½.

320+x =17/2*41 =697/2 = 348.5, x =348.5-320 = 28.5

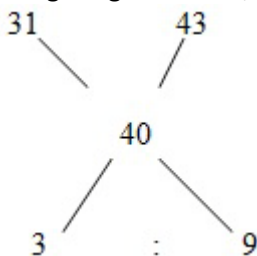14. Exp. P1 = Rs.16 per kg, p2 = Rs. 22 per kg, q1 = 8 kg, q2 = 4 kg

Now , p = (p1q1+p2q2)/(q1+q2)

Average price of the mixture = 8*16+4*22/12 = 128+88/12

= 216 /12 = 18

15. Exp. SP of the mixture = 44, Profit =10%, Then CP = SP*100/110

44*100/110 = Rs. 40 per kg

Using alligation rule, the required ratio =

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   31              43
     \            /
       \        /
         \    /
          40
         /    \
       /        \
     3    :      9
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= 1:3

If 36 kg is 3 part then 1 part is 36*1/3 =12

16. Exp. Total new quantity = original sol + water =66+4 =70

New ratio = 3:2, New quantity of milk =3/5*70 = 42 Lit,

New quantity of water = 2/5*70 = 28 Lit

Water present initially = (28-4) = 24 Lit

Ratio = 42/24 =7/4 There for x = 4

17. Exp. On double- refining impurity decreases from 20% to 4%., but the amount of pure oil in both cases remains constant,

i.e, 96% of double refined oil = 80% of single refined oil.

Let x be the quantity of double-refined oil

Then   96*x/100 = 80*30/100, x = 30*80 /96 = 25

18. Exp. Water in the given mixture = 10*20/100 =2 kg,

And spirit =   (20-2) = 18 kg

Let x kg of water added, Then, x+2/20+x *100 ==25

4x+8 =20+x, or x =4 kg

19. Exp. Suppose the can initially contains 7x and 5x litres mixtures A and B respectively

Quantity of A in mix. left =[7x –7/12 * 9] =[7x –21/4]

Quantity of B in mix. left =[5x –5/12 *9] = [5x - 15/4]

Therefore [7x-21/4]   =7/9 or 28x-21 = 7/9

        [5x-15/4]+9   20x+21

252x -189 =140x+147 or 112x =336 , x = 3.

Quantity of   A in the can initially =7*3 =21

20. Exp. Let the weight of the two alloys be w each

Required ratio =

(3w/12 +4w/18): (2w/12 +11w/18): (7w/12+3w/18)

= 17w/36 : 28w/36 : 27w/36

= 17 : 28 : 27